Structures concurrentes en sémantique des jeux

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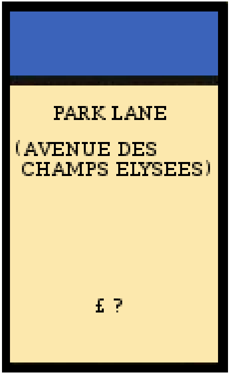
Soutenance de thèse

13 juillet 2017

A Monopoly problem: the theory

Property of Albert.



Property of Barnabé.

To buy both:

$$\text{price} = \text{price Albert} + \text{price Barnabé} \stackrel{?}{=} \text{price Barnabé} + \text{price Albert}$$

A Monopoly problem: the practice

— (to B) How much for park lane?
— (B) **£400**.
— (to A) How much for mayfair?
— (A) Never mind the monopoly, I need money: **£300**

Total price: £700.

A Monopoly problem: the practice

— (to B) How much for park lane?
— (B) **£400**.
— (to A) How much for mayfair?
— (A) Never mind the monopoly, I need money: **£300**

Total price: £700.

— (to A) How much for mayfair?
— (A) I need money: **£300**
— (to B) How much for park lane?
— (B) Eh! This monopoly will cost you: **£600**

Total price: £900.

Beyond formulae : strategies

$$\text{price} = \text{price Albert} + \text{price Barnabé} \stackrel{?}{=} \text{price Barnabé} + \text{price Albert}$$

Albert *then* Barnabé

(to *A*) How much for mayfair?	q_A
(*A*) I need money: **£300**	↓ 300
(to *B*) How much for park lane?	↓ q_B
(*B*) Eh! **£600**	↓ 600
Total price: £900	↓ 900

Beyond formulae : strategies

$$\text{price} = \text{price Albert} + \text{price Barnabé} \stackrel{?}{=} \text{price Barnabé} + \text{price Albert}$$

Albert *then* Barnabé

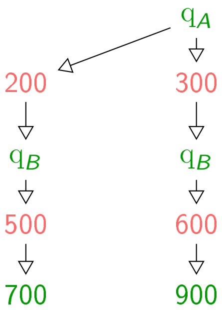
(to A) How much for mayfair?

(A) I need money: **£200**

(to B) How much for park lane?

(B) Eh! **£500**

Total price: £700



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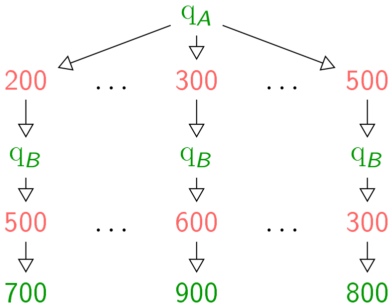
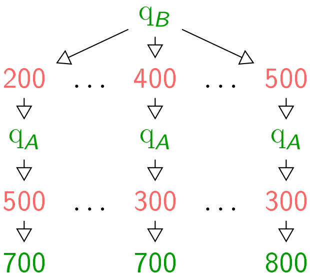


Beyond formulae : strategies

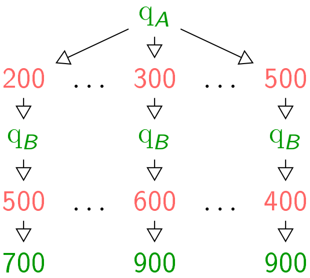
$$\text{price} = \text{price Albert} + \text{price Barnabé} \stackrel{?}{=} \text{price Barnabé} + \text{price Albert}$$

Same *formula*, two **different strategies:**

Barnabé *then* Albert



Albert *then* Barnabé



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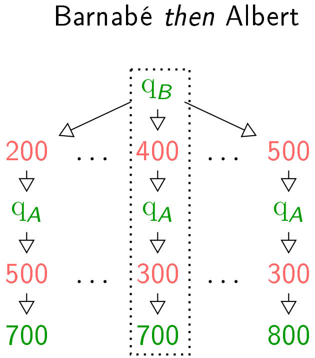


A **branch** is a play of the strategy against a particular environment.

Two kinds of interpretations

Those strategies correspond to different programs:

$x = \texttt{askAlbert}()$	$x = \texttt{askBarnabé}()$
$y = \texttt{askBarnabé}()$	$y = \texttt{askAlbert}()$
$\text{tot} = x + y$	$\text{tot} = x + y$

Two kinds of interpretations

Those strategies correspond to different programs:

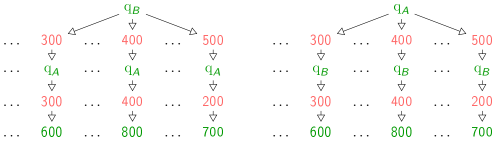
x = `askAlbert()`	x = `askBarnabé()`
y = `askBarnabé()`	y = `askAlbert()`
tot $= x + y$	tot $= x + y$

These programs have two different interpretations (or semantics):

- *Static* interpretation via **formulae** (Input / Output)

$$\text{price} = \text{price Albert} + \text{price Barnabé}$$

- *Dynamic* interpretation via **strategies** (Interactive process)



Game semantics

Game semantics interpretation of more complicated formulae:

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?

q

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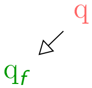

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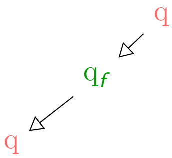

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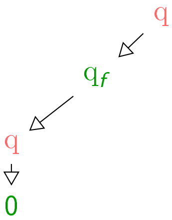

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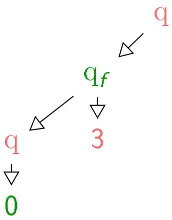
$$\texttt{average}(f) = \frac{3 + f(1)}{2} \qquad \text{against} \qquad f(x) = 3$$



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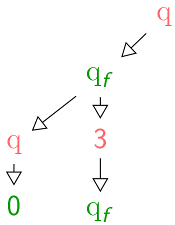

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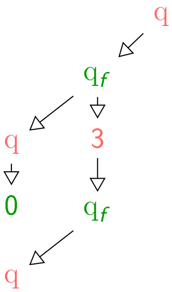

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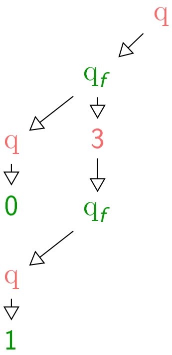

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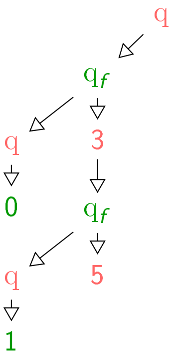
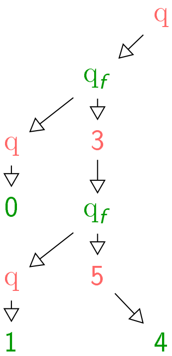
$$\texttt{average}(f) = \frac{3+5}{2} \qquad \text{against} \qquad f(x) = 5$$



Game semantics

Game semantics interpretation of more complicated formulae:

$$\texttt{average}(f) = 4 \qquad \text{against} \qquad f(x) = 2 \times x + 3$$



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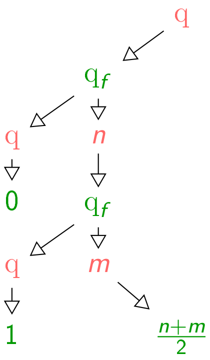

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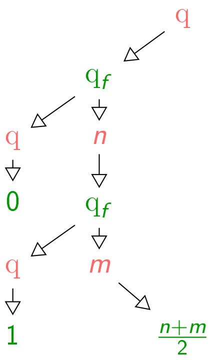
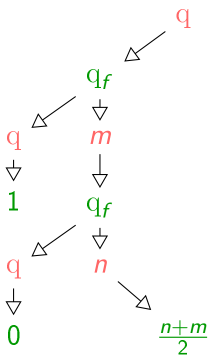


left *then* right



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What if we want to compute in parallel?

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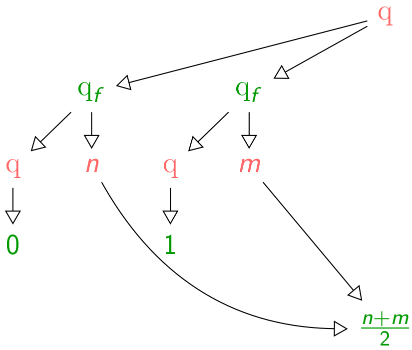

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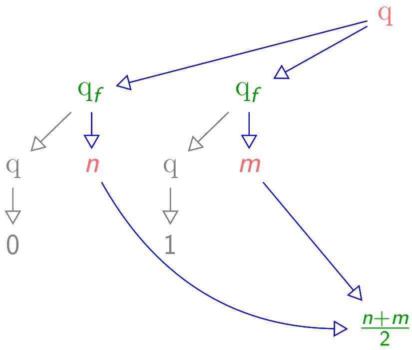

Problem

Usual game semantics only manipulates sequential plays.

Is this sound?

For which strategies is this optimization correct?

Code	
`func` $f_1(x)$: `return` $2x + 3$	

`func` $f_2(x)$:
  `if(rand()) return` $2 \times x$
  `else return 0`	

`func` $f_3(x)$:
  increment counter
  `return counter`	

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Code	Average
`func` $f_1(x)$: `return` $2x + 3$	4
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Correct **sequential** strategies are **innocent** and **well-bracketed**.

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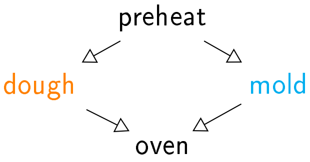
Problem

And what about *concurrent* strategies?

Partial orders or interleavings?

To represent concurrency:



Partial orders or interleavings?

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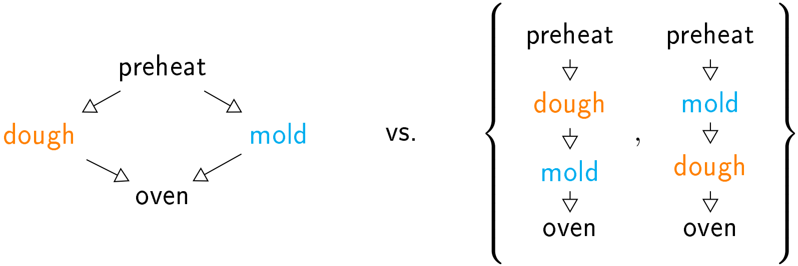


partial order

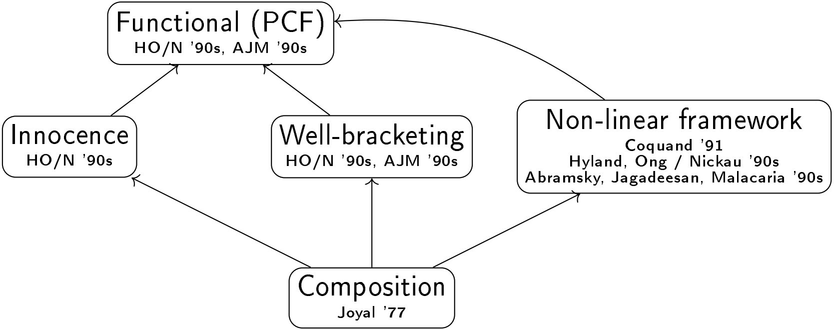
interleavings

(true concurrency)

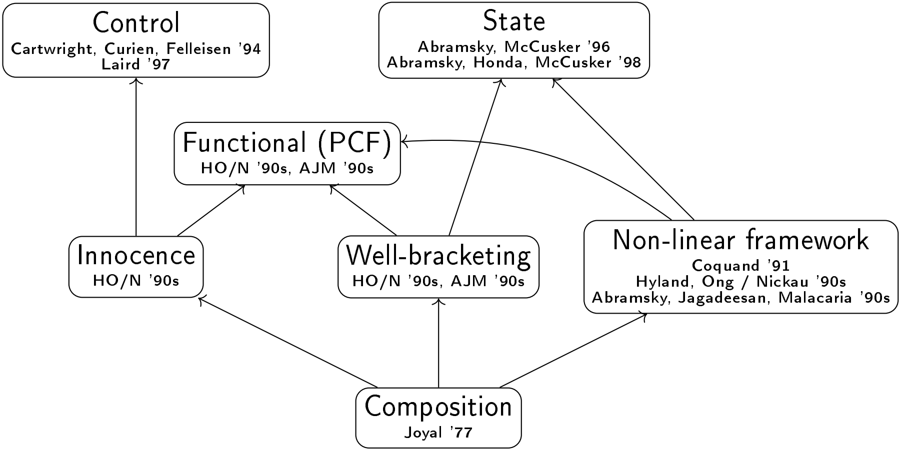
Game semantics: sequential strategies

Composition
Joyal '77

Game semantics: sequential strategies



Game semantics: sequential strategies



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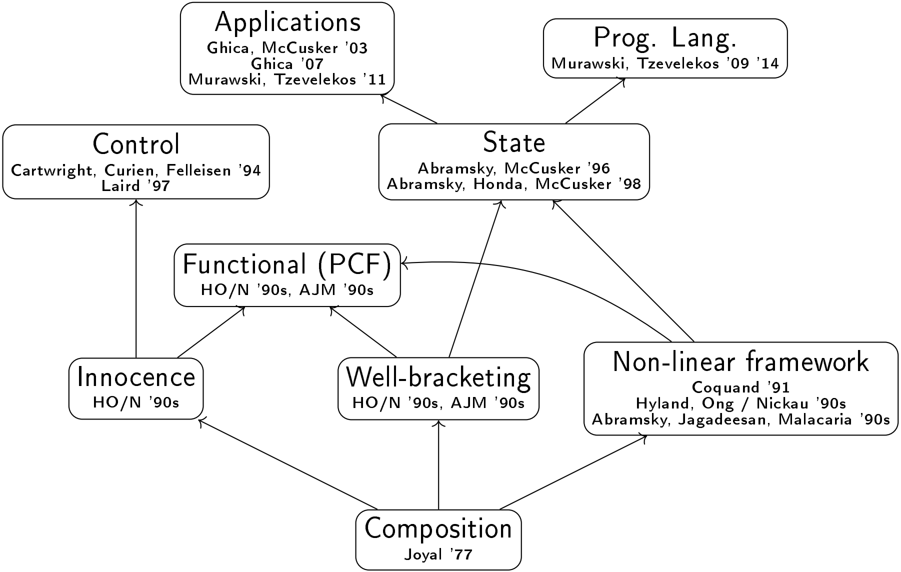
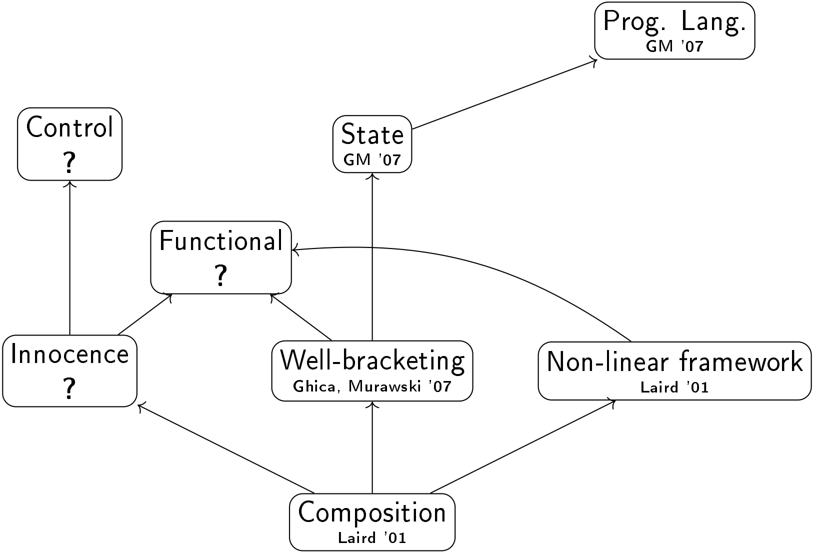
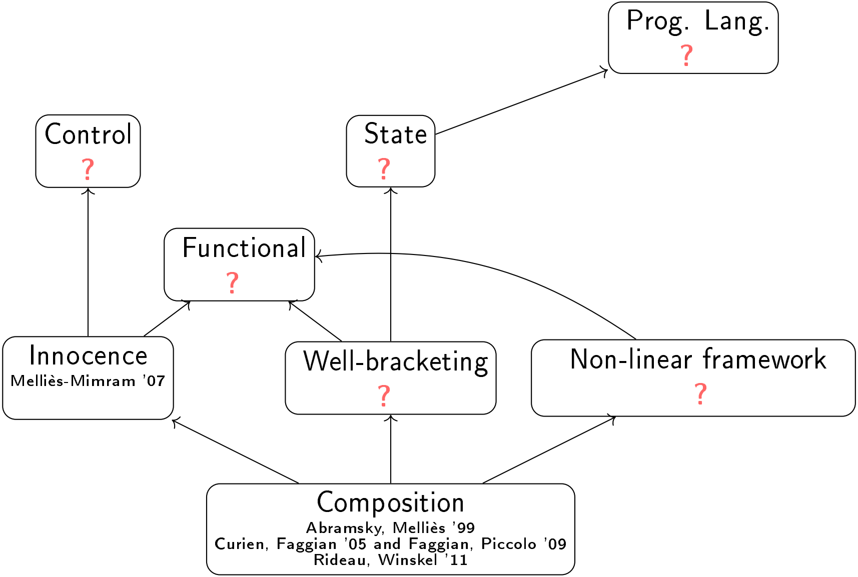


Game semantics: concurrent strategies via interleavings



Game semantics: truly concurrent strategies



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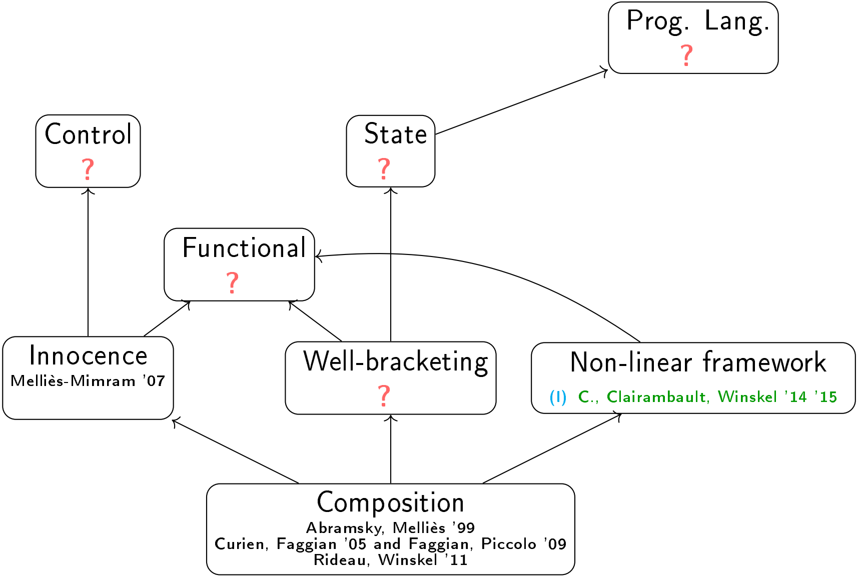

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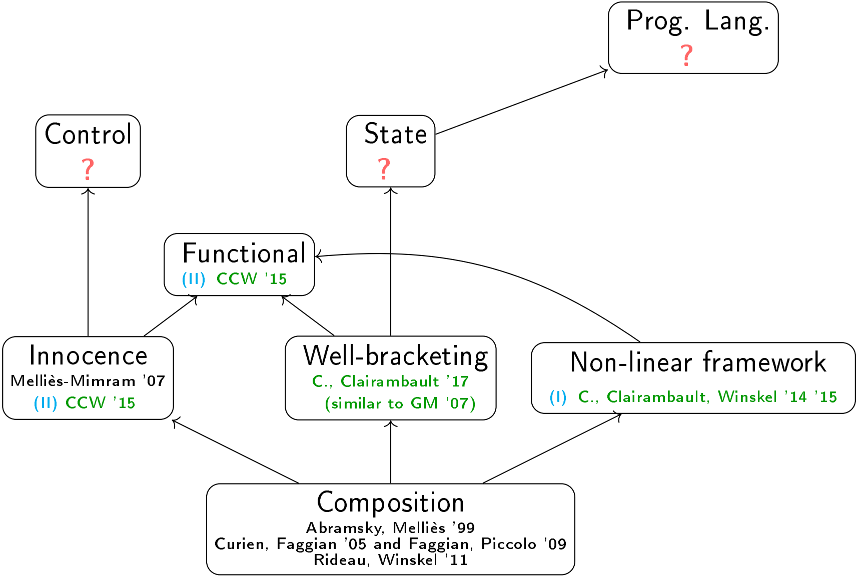

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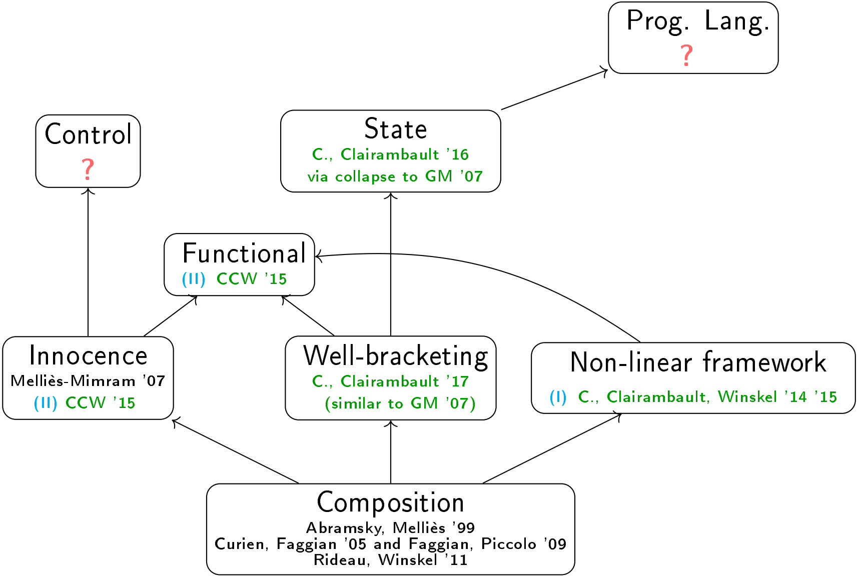

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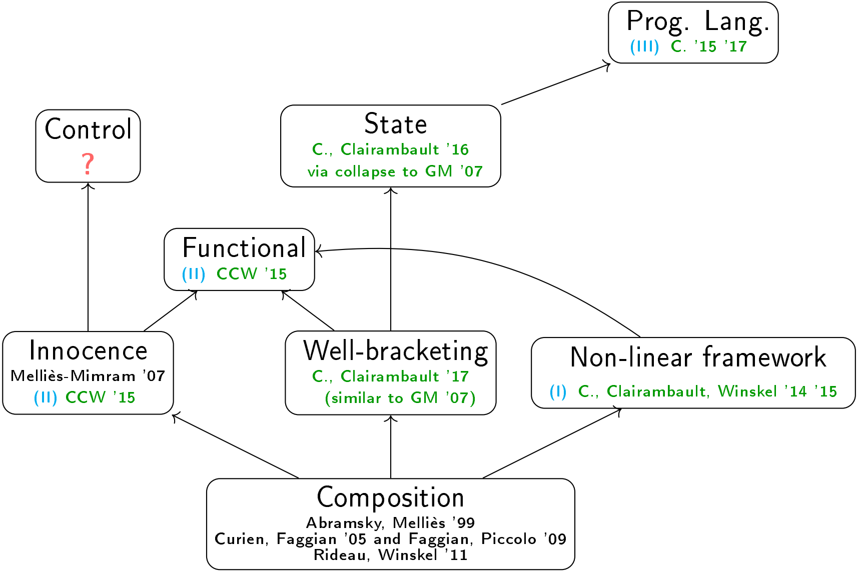
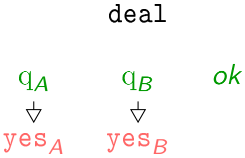


I. A framework for partial-order strategies

Games

Game: partial order where each *move* has a polarity (*Opponent*, *Player*).



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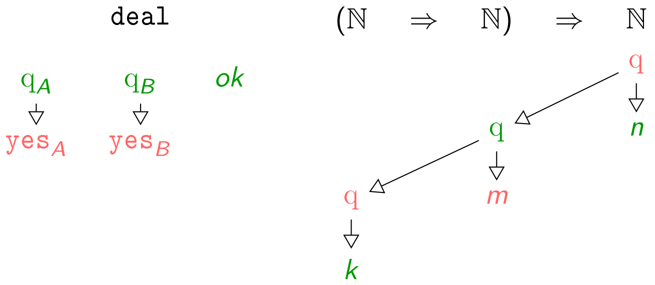

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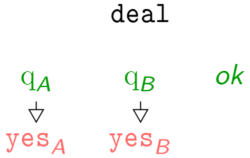
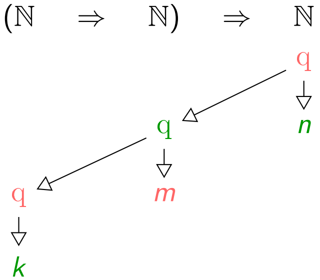
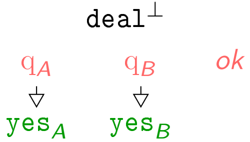
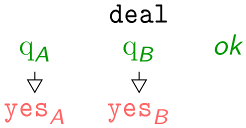




From a game A we build its dual $A^\perp$ by reversing polarities:



(Deterministic) Strategies



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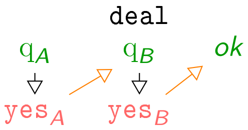
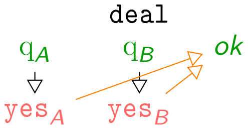


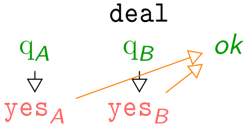
$\sigma_{A;B}$: Albert *then* Barnabé

(Deterministic) Strategies



$\sigma_{A\|B}$: Albert *and* Barnabé *in parallel*

(Deterministic) Strategies



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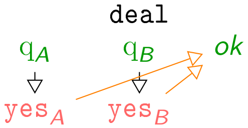
Definition

A **strategy** on $(A, \leq_A)$ is a partial order $\sigma = (S, \leq_S)$ such that

- $S \subseteq A$, and if $s \leq_A s'$ then $s \leq_S s'$ (rule-respecting)
- S only adds immediate links $\ominus \rightarrow \oplus$ (courteous)

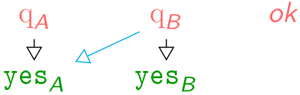
(Deterministic) Strategies



$\sigma_{A\|B}$: Albert *and* Barnabé *in parallel*

Strategies on $A^\perp$ represent **counter-strategies**:

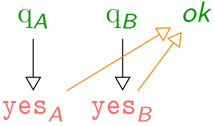
τ : $\mathtt{deal}^\perp$

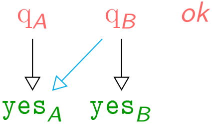


Interaction of strategies

Given a strategy on A and one on $A^\perp$, how do they interact?

$\sigma_{A\|B} : \texttt{deal}$ $\qquad$ $\sigma_{A\|B} \circledast \tau$ $\qquad$ $\tau : \texttt{deal}^\perp$





Interaction of strategies

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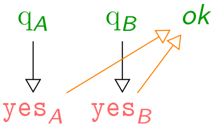
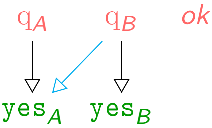
$\sigma_{A\|B}$: deal



$\sigma_{A\|B} \circledast \tau$

τ : deal$^\perp$



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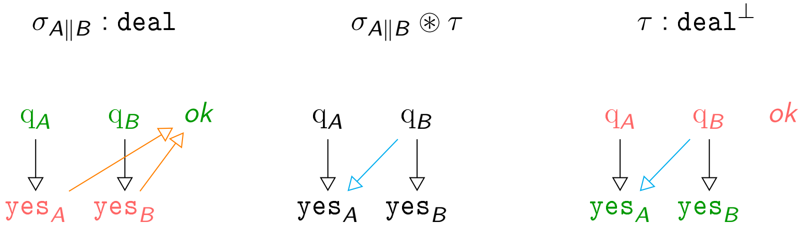

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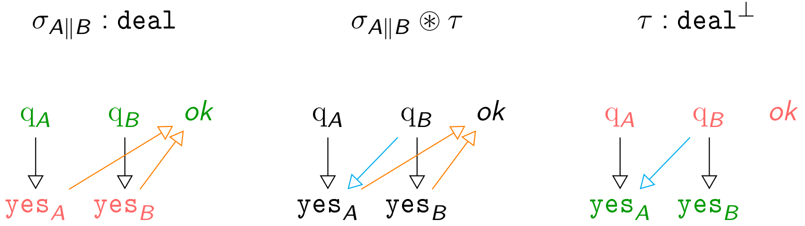

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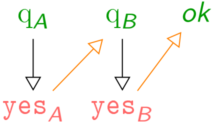
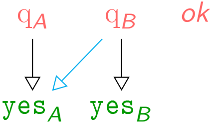
Given a strategy on A and one on $A^\perp$, how do they interact?

$\sigma_{A;B}$: deal $\qquad\qquad \sigma_{A;B} \circledast \tau \qquad\qquad \tau$: $\mathtt{deal}^\perp$





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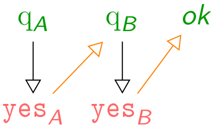
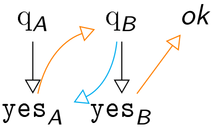
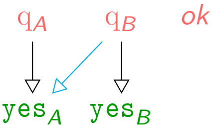
$\sigma_{A;B} : \texttt{deal}$ $\quad$ $\sigma_{A;B} \circledast \tau$ $\quad$ $\tau : \texttt{deal}^\perp$







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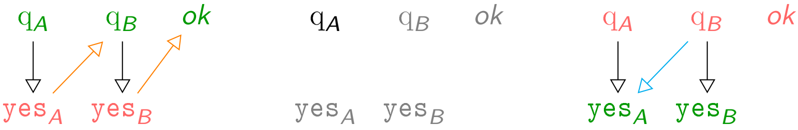

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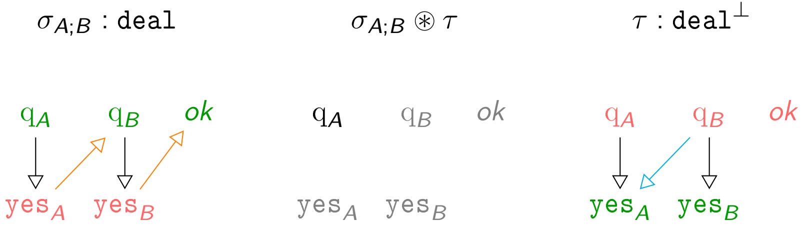


Definition

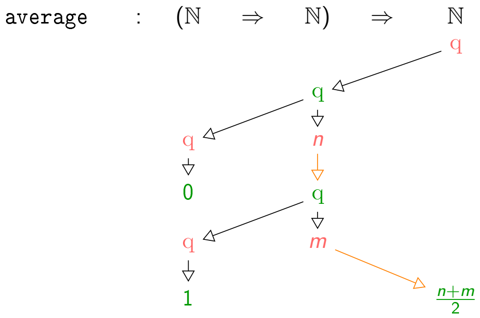
The interaction of $(S, \leq_S)$ and $(T, \leq_T)$ is obtained from

$$(S \cap T, (\leq_S \cup \leq_T)^*)$$

by removing events occurring in a causal loop.

Composition

Interaction is used to compute application and composition of strat.



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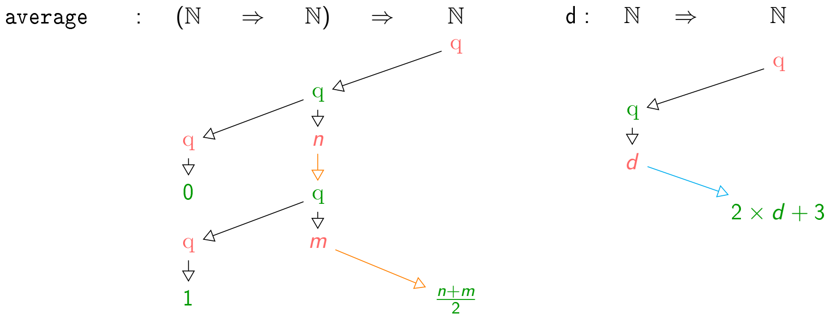

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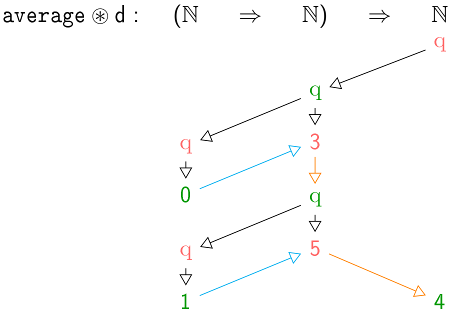

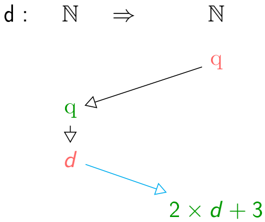

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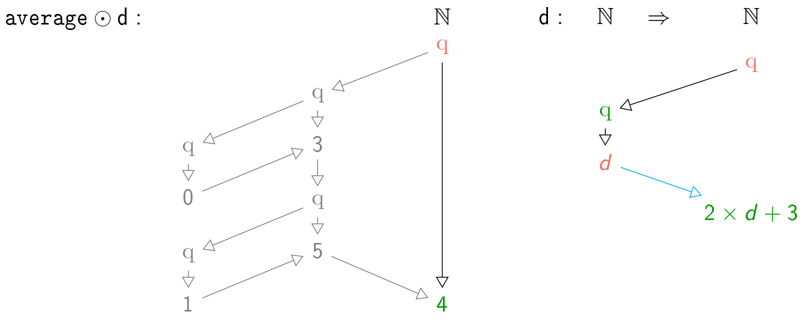
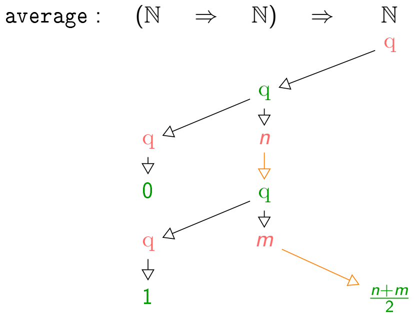


Linearity & duplication

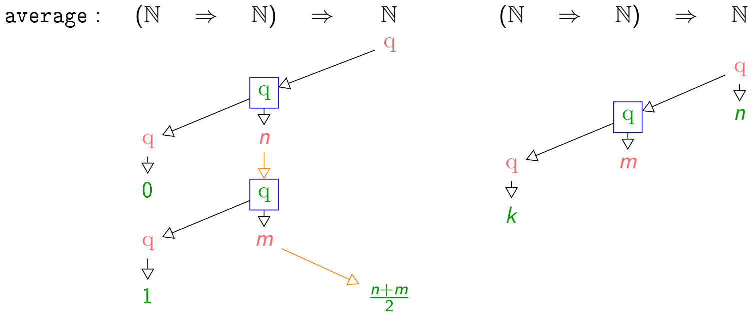
Our average strategy is **not** a strategy!



Linearity & duplication

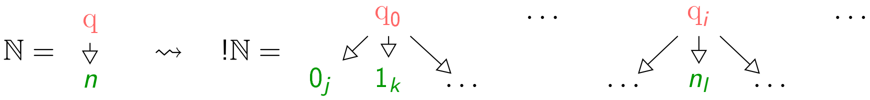
Our average strategy is **not** a strategy! It is not linear:

$$\texttt{average}(f) = \frac{f(0) + f(1)}{2}$$

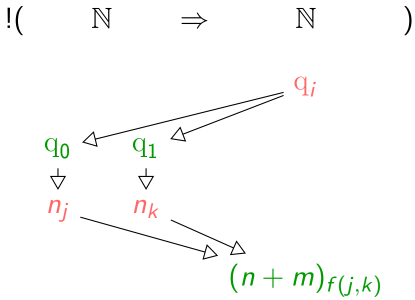


Copy indices & the game $!A$

To solve this problem, we play on expanded arenas.



A (parallel) strategy implementing $d(x) := x + x$ becomes:



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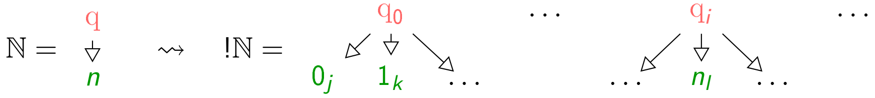

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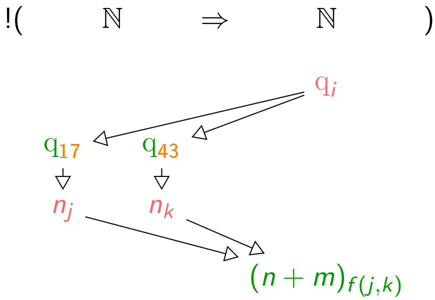

The cartesian-closed category CHO

We get a cartesian closed category (CCC):

Theorem (C., Clairambault, Winskel)

The following structure CHO is a CCC:

- Objects *Games A (which are alternating forests)*
- Morphisms *Strategies uniform (and single-threaded) on* $!(A \Rightarrow B)$.

(Usual sequential innocent HO games form a subcategory of CHO)

$\rightsquigarrow$ Rich semantic framework to interpret concurrent higher-order programs.

What was swept under the rug

- Manage to identify strategies up to **choice of copy indices**.

  $\rightsquigarrow$ A notion of **weak isomorphism**

- Define a notion of **uniformity** (when a strategy is blind to Opponent indices).

  $\rightsquigarrow$ Strategies become equipped with **symmetry**.

- Show that, on uniform strategies, weak isomorphism is a **congruence**.

  $\rightsquigarrow$ Proof of a **bipullback property** of interaction.

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  $\rightsquigarrow$ Proof of a **bipullback property** of interaction.

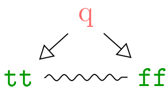
- Representation of **nondeterminism** in strategies

  $\rightsquigarrow$ Addition of **essential events**.

Nondeterminism via event structures

Nondeterminism can be represented via a conflict relation:

`coin` : $\mathbb{B}$



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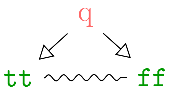
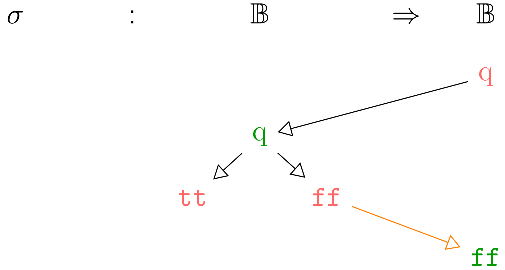


Definition

An **event structure** is a partial order E equipped with a binary relation (representing conflict) satisfying some axioms.

Hidden divergences via essential events

But, nondeterminism and composition require some care:



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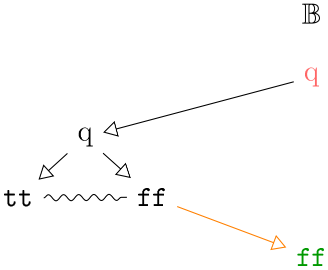

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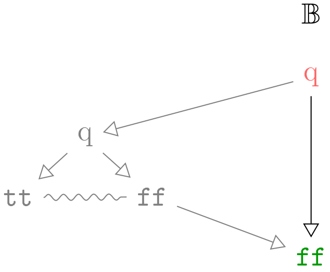

$\sigma \odot \texttt{coin}$ coincides with $\texttt{ff}$!

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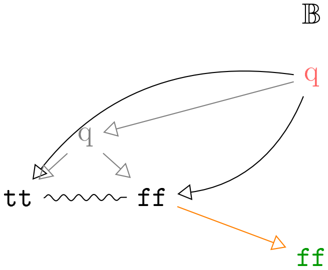

$\sigma \odot$ `coin` coincides with `ff` !

$\tau \circledcirc \sigma$ retains more information than $\tau \odot \sigma$ (must adequacy)

II. Order of evaluation and innocence

PCF and its interpretations

We can interpret PCF in CHO:

$$A, B ::= \mathbb{N} \mid \mathbb{B} \mid \texttt{proc} \mid A \Rightarrow B \qquad \text{(PCF types)}$$

$$M, N ::= \texttt{tt} \mid \texttt{ff} \mid \texttt{if}\ M\ N_1\ N_2 \mid () \mid M; N$$

$$\mid x \mid \lambda x.\, M \mid M\, N \mid Y \mid \dots \qquad \text{(PCF terms)}$$

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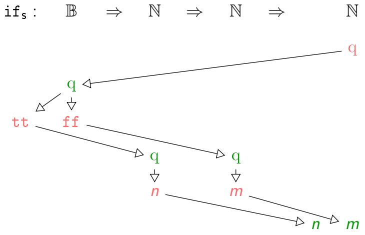
Two interpretations of `if` in CHO:



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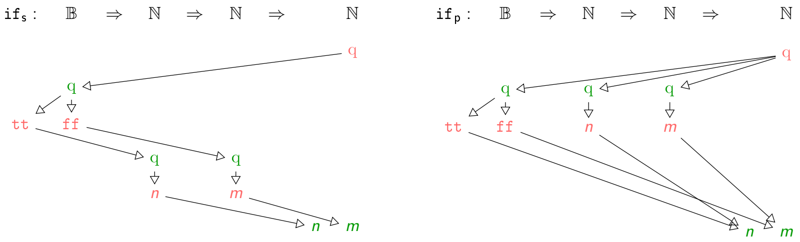
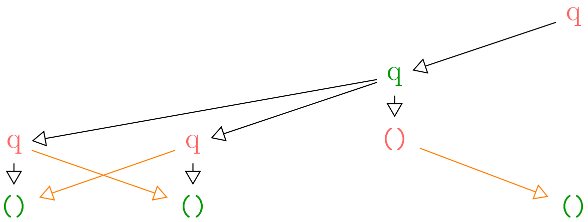


These two interpretations are indistinguishable by terms of PCF.

Sometimes `parallel` if just is not the same

$\texttt{sync}: \quad (\texttt{proc} \Rightarrow \texttt{proc} \Rightarrow \texttt{proc}) \Rightarrow \texttt{proc}$



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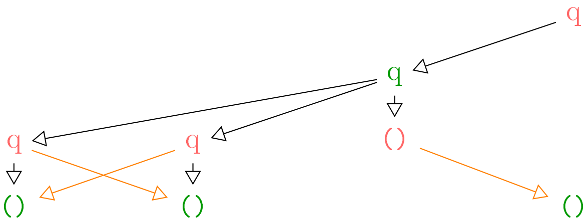
sync : (proc ⇒ proc ⇒ proc) ⇒ proc



sync can distinguish both implementations of `if`.

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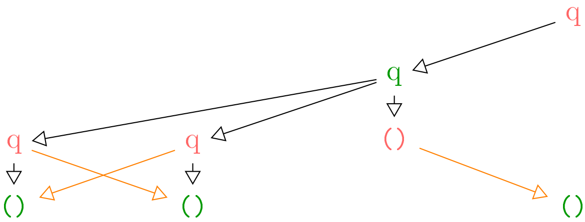

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(For $M_\epsilon = \lambda x \lambda y.\mathrm{if}_\epsilon\,(x;\mathtt{tt})\,(y;\mathtt{tt})\,\mathtt{tt}$:

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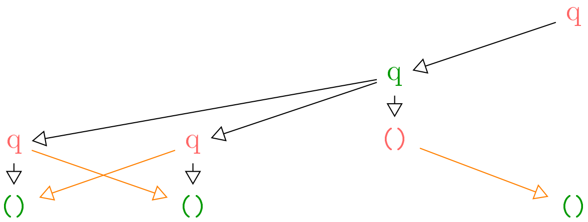

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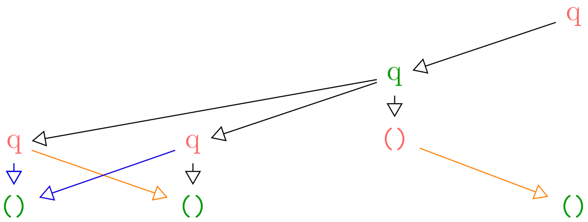

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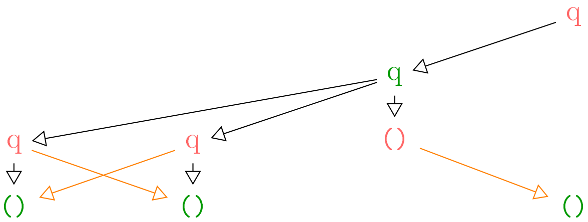

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Banning such patterns gives a notion of **concurrent innocence**.

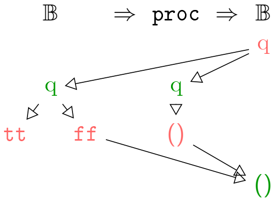
Finite definability

A **PCF strategy** is an innocent and well-bracketed strategy.

Theorem (Finite definability)

If σ is PCF strategy, there exists a term M of PCF such that

$$[\![M]\!] \text{ and } \sigma \text{ are indistinguishable by PCF strategies.}$$



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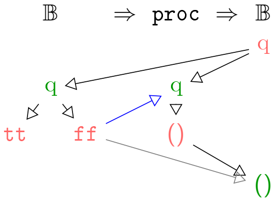

$\rightsquigarrow \qquad \lambda b \lambda x.\, \mathrm{if}\, b \perp x$

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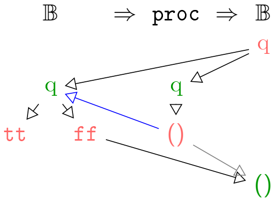

$\rightsquigarrow \quad \lambda b \lambda x.\, x;\ \text{if}\ b \perp ()$

What was swept under the rug 2

- Innocence and well-bracketing:
  - Stability under **composition**.
    $\rightsquigarrow$ **Forking** lemma.
  - Requires the addition of **visibility** (and **locality**).
    $\rightsquigarrow$ Control **interferences** between Player threads

What was swept under the rug 2

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    $\rightsquigarrow$ **Forking** lemma.
  - Requires the addition of **visibility** (and **locality**).
    $\rightsquigarrow$ Control **interferences** between Player threads
- Finite definability:
  - Define a notion of **finite** strategies.
    $\rightsquigarrow$ **Reduced form** (P-view "dag")
  - **Factorisation** theorem for higher-order strategies.
    $\rightsquigarrow$ First-order / λ-calculus

III. What lies beyond PCF?

What real concurrent programs are made of

```
while(1) {                                  ||  while(1) {
  while (canWrite == 0) ;                   ||    while (canRead == 0) ;
  x = produce();                            ||    x = value;
  value := x;                               ||    consume(x);
  canRead := 1; canWrite := 0;              ||    canRead := 0; canWrite := 1;
}                                           ||  }
```

$\rightsquigarrow$ loops, conditionals, function calls, shared memory.

What real concurrent programs are made of

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$\rightsquigarrow$ $\underbrace{\text{loops, conditionals, function calls}}_{\text{in PCF}}$, shared memory.

What real concurrent programs are made of

$$\begin{array}{l} \texttt{value} := 1; \\ \texttt{canRead} := 1; \end{array} \Bigg\| \begin{array}{l} f \leftarrow \texttt{canRead}; \\ x \leftarrow \texttt{value}\ ; \end{array}$$

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What real concurrent programs are made of

$$\begin{array}{l} \texttt{value} := 1; \\ \texttt{canRead} := 1; \end{array} \;\Big\|\; \begin{array}{l} f \leftarrow \texttt{canRead}; \\ x \leftarrow \texttt{value}\ ; \end{array}$$

Expectation: $f = 1$ implies $x = 1$

$\rightsquigarrow$ loops, conditionals, function calls, shared memory

Running concurrent programs on a processor

```
value := 1;
canRead := 1;
```

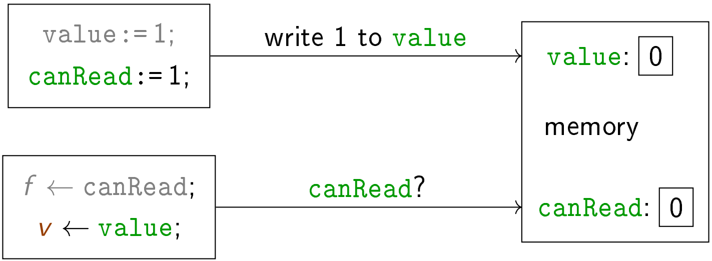
$f \leftarrow$ `canRead`;
$v \leftarrow$ `value`;

`value`: 0

memory

`canRead`: 0

Running concurrent programs on a processor



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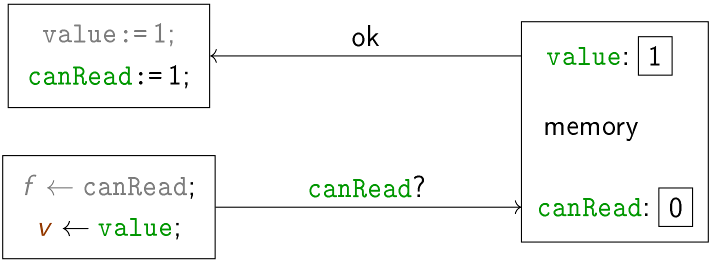

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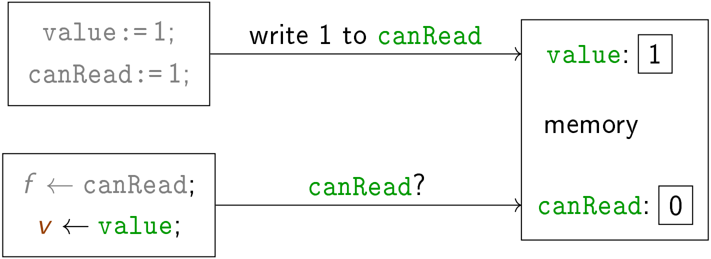

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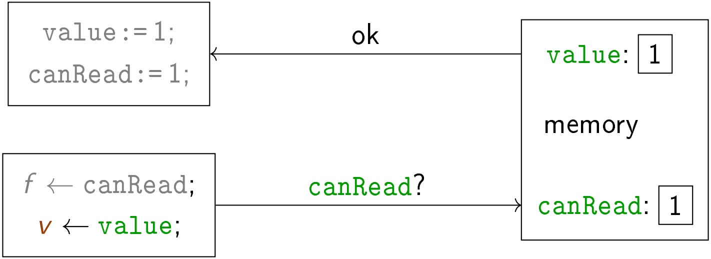

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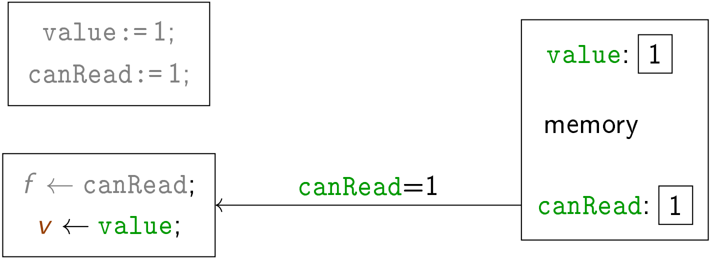

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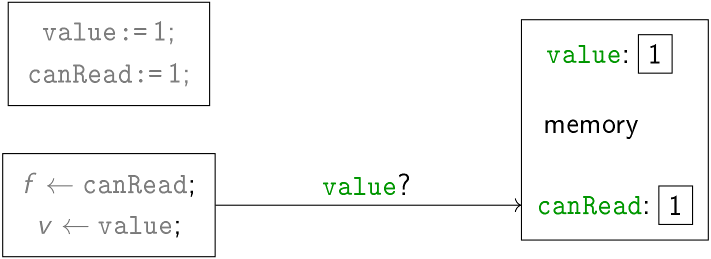

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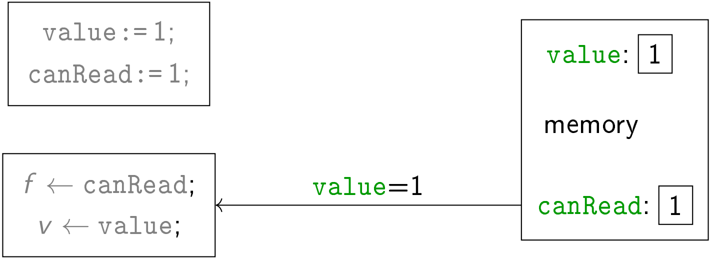


Running concurrent programs on a processor

```
value := 1;
canRead := 1;
```

$f \leftarrow$ `canRead`;
$v \leftarrow$ `value`;

`value`: 1

memory

`canRead`: 1

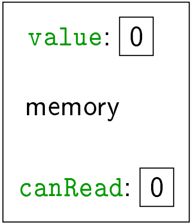
Several executions, but never $f = 1$ and $v = 0$.

Weak memory models

On some processors, we get instead:

```
value := 1;
canRead := 1;
```

```
f ← canRead;
v ← value;
```



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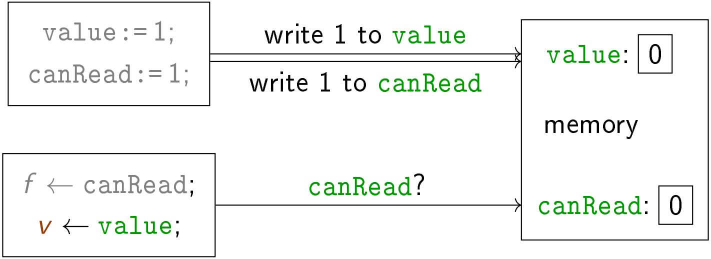

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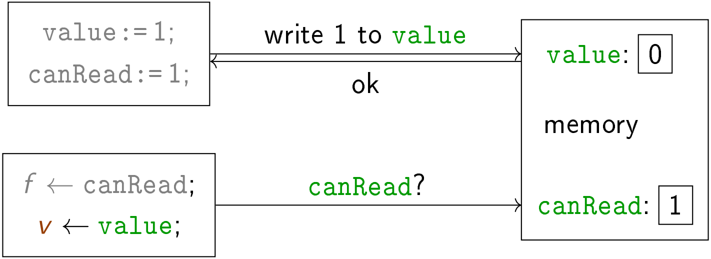

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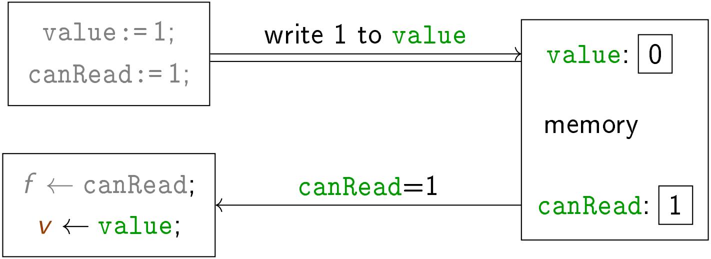

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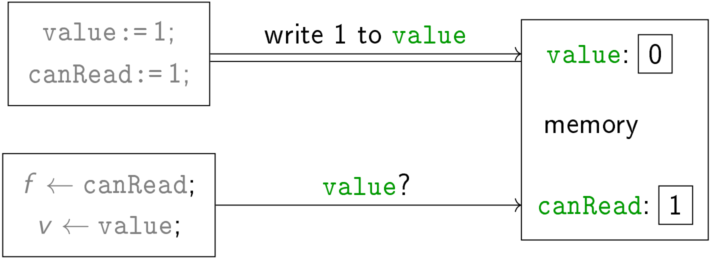

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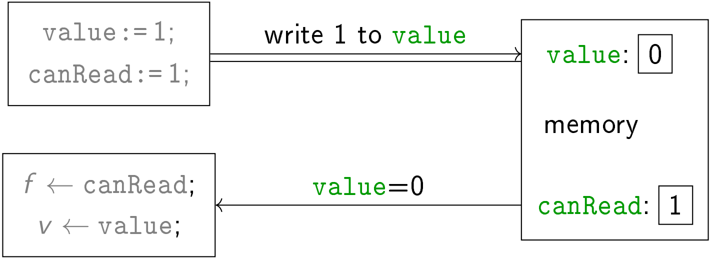

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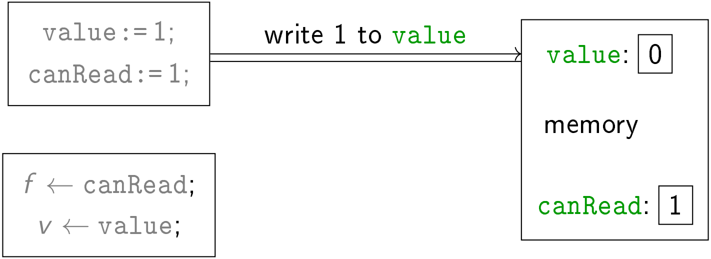

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Goal: Model *denotationally* such **complex** reorderings.

Taking a closer look

```
x := 1;
y := 1;
r ← z;
```

x:	?
z:	?
y:	?

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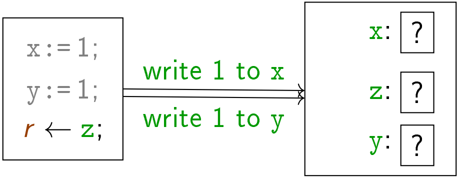

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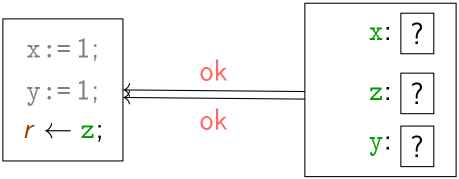

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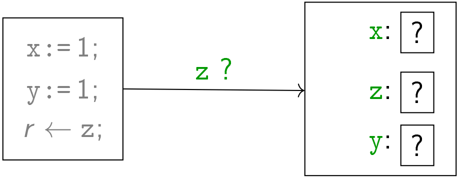

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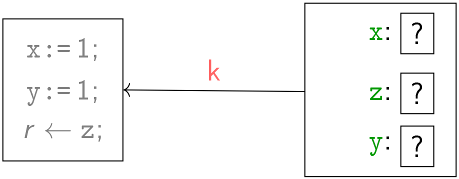

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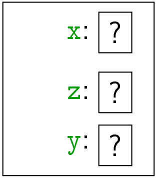
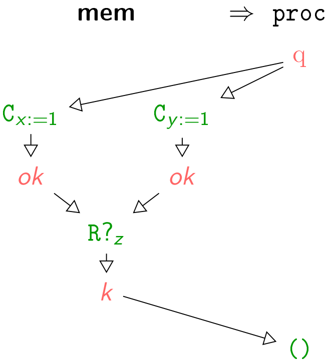


The behaviour of the thread corresponds to a strategy:



Modelling weak memory models

An old motto (Reynolds, implemented in game semantics by Abramsky and McCusker)

The behaviour of imperative programs can be described as the interaction of a (deterministic) pure functional program interacting with a memory.

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Individual programs are interpreted as **parallel** strategies

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Implementation of seq (here for PSO):

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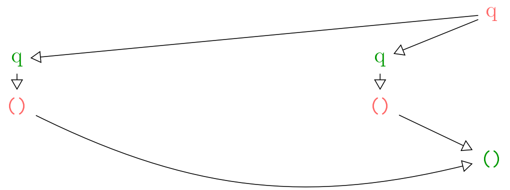

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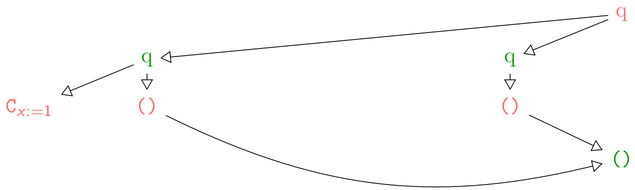

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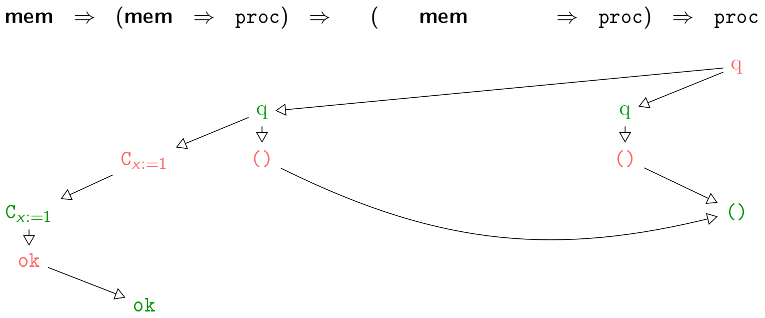

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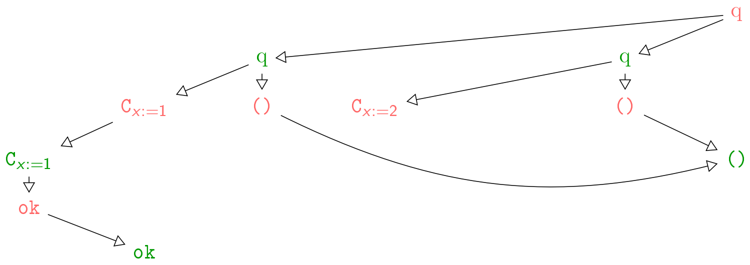

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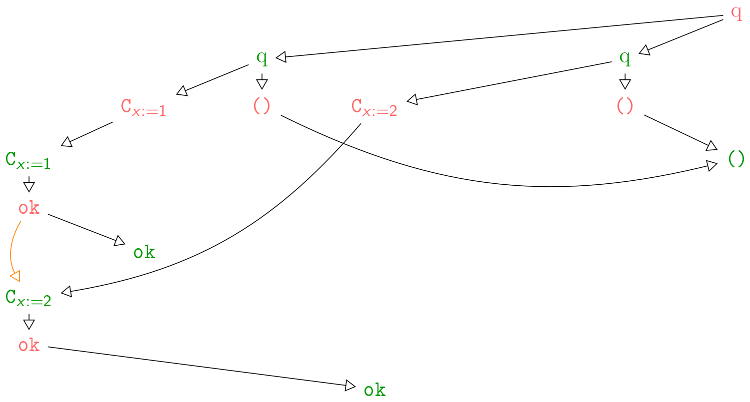

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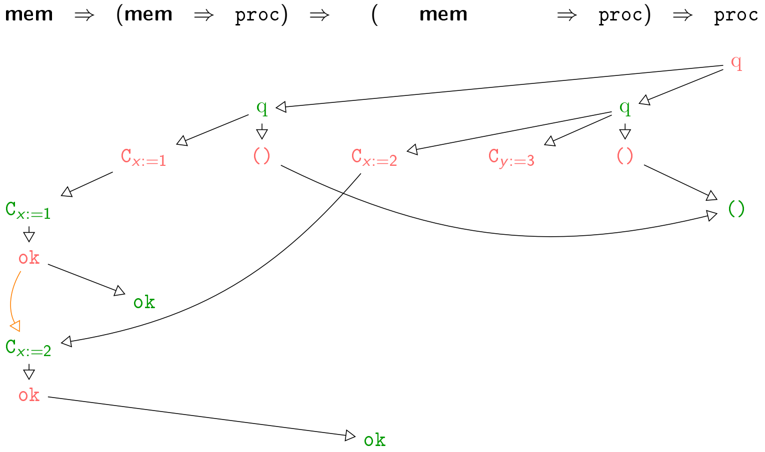

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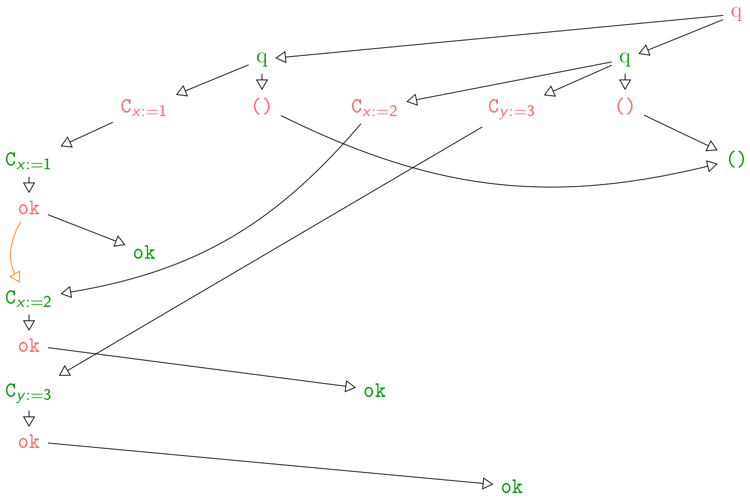

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Depends on the architecture $\mathcal{A}$:

- Thread operations: $\texttt{seq}_{\mathcal{A}}$, $\texttt{read}_{\mathcal{A}}$, $\texttt{write}_{\mathcal{A}}$ implement the reorderings specific to $\mathcal{A}$.
- Memory (representing caches, barriers, ...) is represented by

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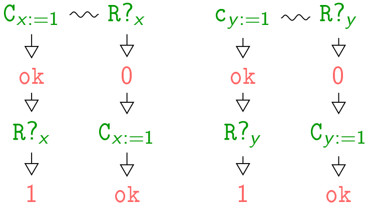

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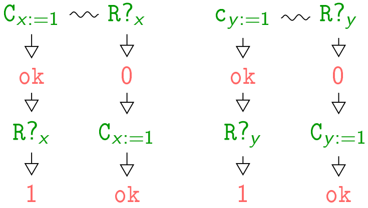

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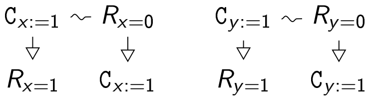

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Perspectives

- Describe finitely non-innocent strategies.

  $\rightsquigarrow$ Represent efficiently operations on them.

- Which language corresponds to innocent concurrent strategies?

  $\rightsquigarrow$ Concurrent control operators (eg. `fork`).

- Weaker architectures (eg. ARM) and software specifications.

  $\rightsquigarrow$ How to handle speculation, complex barriers specification.