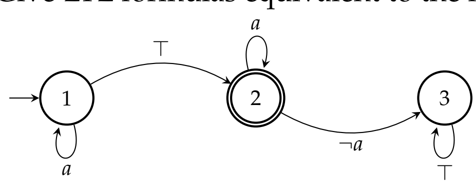
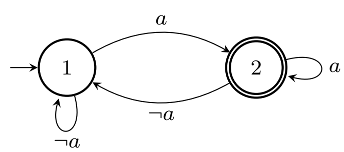
TD4: GNBAs

simon.castellan@ens-lyon.fr

***Exercise* 1** (Automata)**. —**

1. Give NBAs recognizing the same language as the following formulas:
   - $\mathbf{G}(a \to Fa)$
   - $a\mathbf{W}b = \mathbf{G}a \vee (a\mathbf{U}b)$
2. For $a\mathbf{W}b$, build the corresponding GNBA and compare.
3. Give LTL formulas equivalent to the following automata:





***Exercise* 2** (LTL versus NBAs)**. —** Consider the language $L = \{q_0 \dots q_n \mid a \in q_{2i}\}$ on the set of atomic formulae $\mathrm{At} = \{a\}$.

1. Give a NBA recognizing L
2. Prove that for any LTL-formula φ, there exists $n \in \mathbb{N}$ such that $\{a\}^{n+1}\emptyset\{a\}^{\omega} \models \varphi$ if and only $\{a\}^{n+2}\emptyset\{a\}^{\omega} \models \varphi$.
3. Prove that NBAs are strictly more expressive than LTL formulas.
4. Show that $[\alpha] \mapsto \mathbf{XX}\alpha \wedge a$ has no greatest fixpoints. Deduce that the lattice of LTL formulae is not complete.

***Exercise* 3** (Into the past)**. —** We write At for the set of atomic propositions.

1. We wish to extend the syntax of LTL with two more modalities $\varphi\mathbf{S}\psi$ ("since") and $\mathbf{P}\varphi$ ("previous"). Explain how we could extend the semantics of LTL to accomodate those new constructors.
2. Explain how to define connectors to capture the informal properties: "at some point in the past" and "always in the past".
3. Show how to translate a formula φ of LTL+past operators in φ' where the past operators are not nested by augmenting the set of atomic formulas to At'. How to state correctness our our translation?
4. Show how to reduce a formula of LTL+past operators φ into φ' without past operators, augmenting the set of atomic propositions to At', that is correct in the sense of the previous question.