TD5: CTL

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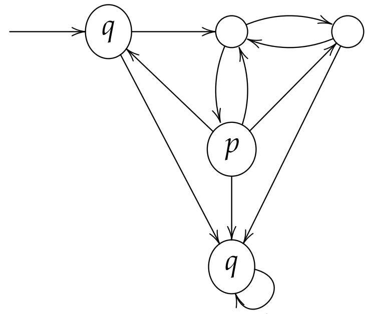
***Exercise* 1** (Warmup)**. —** Consider the following transition system $\mathcal{T}$ ($\mathrm{At} = \{p, q\}$):



Determine which of the following formulas are satisfied by $\mathcal{T}$:

$\mathbf{EF}p$ $\mathbf{AG}p$ $\mathbf{AXEF}p$ $\mathbf{EXEF}p$ $\mathbf{E}(q\mathbf{U}\mathbf{XA}p)$ $\mathbf{A}(q\mathbf{U}\mathbf{XA}p)$

***Exercise* 2** (CTL equivalences)**. —** Two formulas of CTL are equivalent when they are satisfied by the same LTSs. Show the following equivalences:

1. $\mathbf{AF}\varphi \equiv \neg\mathbf{EG}\varphi$
2. $\mathbf{AF}\varphi \equiv \varphi \vee \mathbf{AXAF}\varphi$
3. $\mathbf{AG}\varphi \equiv \varphi \wedge \mathbf{AXAG}\varphi$

***Exercise* 3** (Comparing LTL and CTL)**. —**

1. Are the LTL formula $\mathbf{GF}p$ and the CTL formula $\mathbf{AGAF}p$ equivalent?
2. Are the LTL formula $\mathbf{FG}p$ and the CTL formula $\mathbf{AFAG}p$ equivalent?
3. Are the LTL formula $\mathbf{G}(p \to \mathbf{F}q)$ and the CTL formula $\mathbf{AG}(p \to \mathbf{AF}q)$ equivalent?
4. Are the LTL formula $\mathbf{F}(p \wedge \mathbf{X}p)$ and the CTL formula $\mathbf{AF}(p \wedge \mathbf{AX}p)$ equivalent?

***Exercise* 4** (Difference of expressivity)**. —**

1. Show that there exists a formula φ of LTL such that there exists no CTL formula ψ suh that for all states s of all LTS, $s \models \varphi$ if and only if $s \models \psi$.
2. Show that there exists a formula φ of CTL such that there exists no LTL formula ψ suh that for all states s of all LTS, $s \models \varphi$ if and only if $s \models \psi$.

***Exercise* 5** (LTL is not complete)**. —** Recall the function $g([\alpha]) = [\mathbf{XX}\alpha \wedge a]$ on $\mathrm{At} = \{a\}$ and the language $L = \{w \in \Sigma^\omega \mid \forall i, w_{2i} \neq \emptyset\}$ which is known not to be LTL-definable (see previous TD).

Assume the lattice $LTL/\equiv$ is complete.

1. Show $[\mathbf{G}a] \leq [g([\mathbf{G}a])]$ and $\mathrm{Words}(\mathbf{G}a) \subseteq L$.
2. Let $\varphi = \bigvee_{n\in\mathbb{N}} g^k([\mathbf{G}a])$. Show that $\mathrm{Words}(\varphi) = L$.
3. Conclude.