TD6: Révisions

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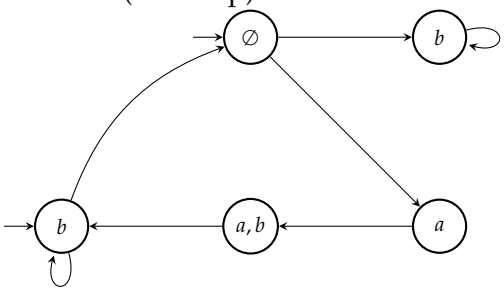
***Exercise* 1** (Warmup)**.** — Consider the following LTS:



For each of the following formula, calculate the saturation set and decide whether this LTS satisfies them:

1. $\mathbf{A}(a\mathbf{U}b) \vee \mathbf{EX}(\mathbf{AG}b)$
2. $\mathbf{AGA}(a\mathbf{U}b)$
3. $(a \wedge b) \to \mathbf{EGEXA}(b\mathbf{U}a)$

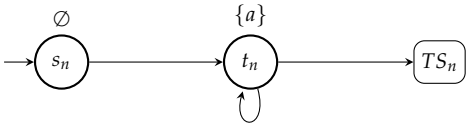
***Exercise* 2** (Mononicity of operators)**.** — Consider a formula of LTL $\varphi(p)$ containg one free variable representing an arbitrary formula, eg. $\mathbf{X}(p) \wedge a$. This defines a function $p \mapsto \varphi(p)$ from $LTL/\equiv$ to itself.

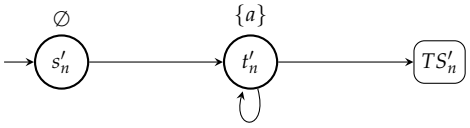
Show that φ is monotone.

***Exercise* 3** (Weak Until in CTL)**.** — We have seen the operator $\mathbf{W}$ in LTL defined as $\varphi\mathbf{W}\psi = \mathbf{G}\varphi \vee (\varphi\mathbf{U}\psi)$ as the greatest fixpoint of $\alpha \mapsto \psi \vee (\varphi \wedge \mathbf{X}\alpha)$.

1. Give an interesting operator in CTL which has the CTL formula $\mathbf{E}(\varphi\mathbf{U}\psi)$ for least fixpoint.
2. Define in CTL its greatest fixpoint that we write $\mathbf{E}(\varphi\mathbf{W}\psi)$.
3. What about $\mathbf{A}(\varphi\mathbf{W}\psi)$?

***Exercise* 4** (Non-equivalence of CTL & LTL)**.** — Recall the LTS TS_n and TS'_n:





Show that for all CTL formula φ, there exists an integer $n \in \mathbb{N}$ such that for all $k \geq n$, we have

$$s_k \models \varphi \Leftrightarrow s'_k \models \varphi \Leftrightarrow s_n \models \varphi \quad \text{and} \quad t_k \models \varphi \Leftrightarrow t'_k \models \varphi \Leftrightarrow t_n \models \varphi$$