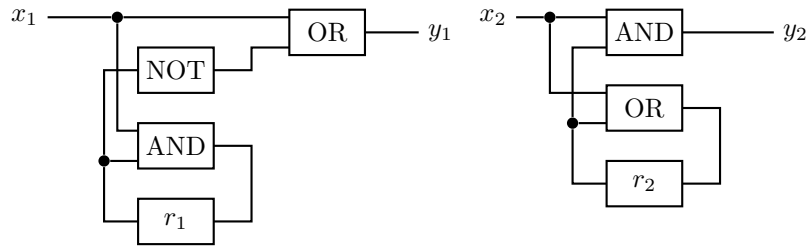


TD de Sémantique et Vérification  
**I– Modelling Concurrent Systems**

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**Exercise 1.**

Consider the following two sequential hardware circuits:



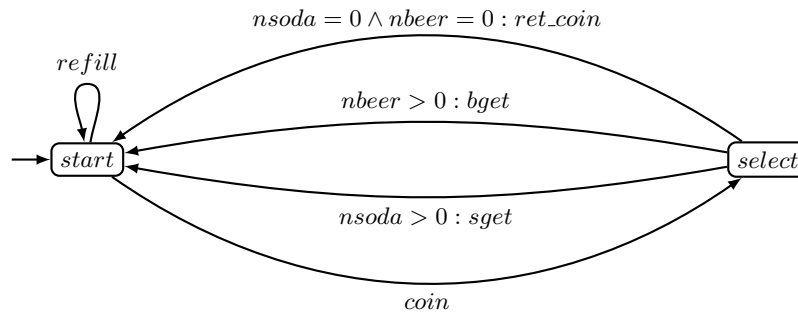
1. Assuming that the initial values of the registers are  $r_1 = 0$  and  $r_2 = 1$ , give the transition systems of both hardware circuits.
2. Determine the reachable part of the interleaving of these transition systems.

**Exercise 2.**

Consider the following Program Graph for a beverage vending machine with internal variables  $\text{Var} := \{nsoda, nbeer\}$  with domain  $\{0, 1, \dots, max\}$ :

$$PG := \langle \text{Loc}, \text{Act}, \text{Effect}, \hookrightarrow, \text{Loc}_0, g_0 \rangle$$

$$\begin{aligned} \text{Loc} &:= \{start, select\} & \text{Act} &:= \{bget, sget, coin, ret\_coin, refill\} \\ \text{Loc}_0 &:= \{start\} & g_0 &:= (nsoda = max \wedge nbeer = max) \\ \text{Effect}(bget, \eta) &:= \eta[nbeer \leftarrow nbeer - 1] & \text{Effect}(coin, \eta) &:= \eta \\ \text{Effect}(sget, \eta) &:= \eta[nsoda \leftarrow nsoda - 1] & \text{Effect}(ret\_coin, \eta) &:= \eta \\ \text{Effect}(refill, \eta) &:= \eta[nsoda \leftarrow max, nbeer \leftarrow max] & & \end{aligned}$$



Give the reachable part of the transition system associated with  $PG$ , when  $max = 2$ .

**Exercise 3.**

Consider a stack of nonnegative integers with capacity  $n$  (for some fixed  $n$ ).

1. Give a transition system representation of this stack. You may abstract from the values on the stack and use the operations *top*, *pop*, and *push* with their usual meaning.
2. Sketch a transition system representation of the stack in which the concrete stack content is explicitly represented.

**Exercise 4.**

Show that the handshaking operator  $\parallel$  that forces two transition systems to synchronise over a subset of their common actions is associative. That is, show that

$$(TS_1 \parallel TS_2) \parallel TS_3 = TS_1 \parallel (TS_2 \parallel TS_3)$$

where  $TS_1, TS_2, TS_3$  are arbitrary transition systems.