ENS DE LYON

TD de Sémantique et Vérification

V– Liveness

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In this sheet, we write $\Sigma = 2^{\mathrm{AP}}$ for some (finite) set AP of atomic propositions.

Exercise 1.
(Warmup) Show that for a LT property P, the following are equivalent:

1. Every finite word $w \in \Sigma^*$ can be completed in a word $w \cdot \sigma \in P$
2. The closure of P is Σ^ω
3. P is dense for the usual topology on Σ^ω (remember that, for a topological space X, $P \subseteq X$ is **dense** when it intersects any non-empty open set).

In that case P is a liveness property. Give examples of liveness conditions.

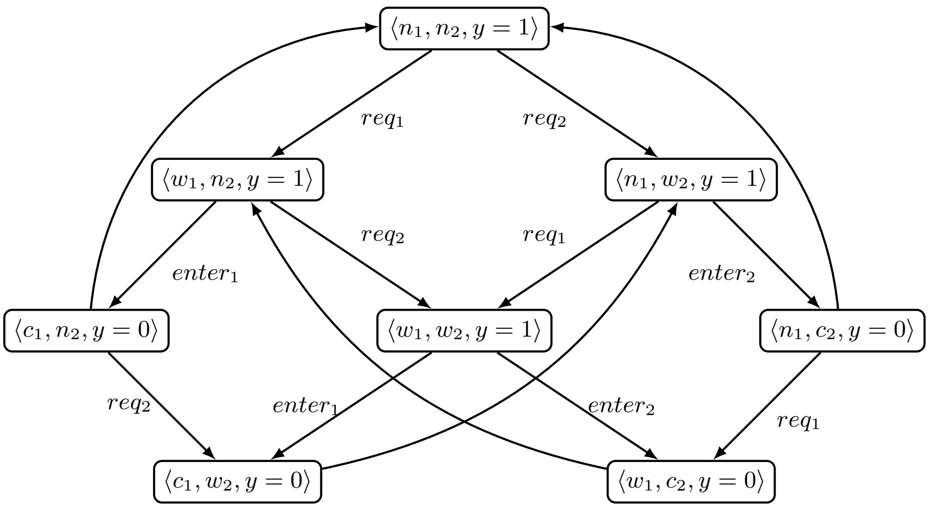
Exercise 2.
Is the union of liveness properties a liveness property? The intersection? Prove it or provide counterexamples.

Exercise 3.
Show the following properties of $cl(\cdot)$ for $A, B \subseteq \Sigma^\omega$

1. $cl(A) \subseteq cl(B)$ when $A \subseteq B$
2. $cl(cl(A)) = cl(A)$
3. $cl(A \cup B) = cl(A) \cup cl(B)$

From those properties, deduce a new proof of the decomposition of any LT property into a liveness property and a safety property.

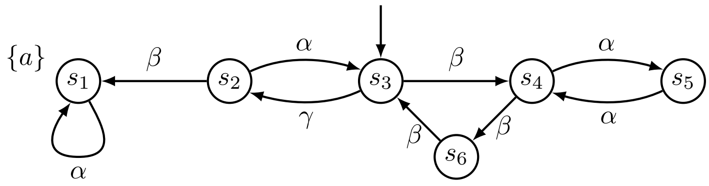
Exercise 4.
Consider the following LTS representing mutual exclusion by means of a semaphore (NB: back arrows are labelled with rel – not drawn on the picture to avoid clutter).



What is a good fairness assumption to ensure that both processes get in the critical section infinitely often in a fair trace?

Exercise 5.

Consider the following transition system TS with the set of atomic propositions $\{a\}$:

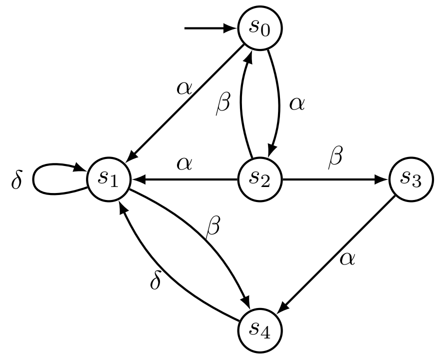


Let the fairness assumption $\mathcal{F}$ be $(\emptyset, \{\{\alpha\}, \{\beta\}\}, \{\{\beta\}\})$. Determine whether $TS \models_{\mathcal{F}}$ "eventually a". Justify your answer!

Exercise 6.

Let TS be a LTS. A $\mathcal{F}$ fairness assumption is **realizable** for TS when for each state s of TS, there exists a fair path starting from s.

Consider the following transition system TS (without atomic propositions):



Decide which of the following fairness assumptions $\mathcal{F}_i$ are realizable for TS. Justify your answers!

1. $\mathcal{F}_1 = (\{\{\alpha\}\}, \{\{\delta\}\}, \{\{\alpha, \beta\}\})$
2. $\mathcal{F}_2 = (\{\{\delta, \alpha\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$
3. $\mathcal{F}_3 = (\{\{\alpha, \delta\}, \{\beta\}\}, \{\{\alpha, \beta\}\}, \{\{\delta\}\})$

Exercise 7.

Show that, given a fixed realizable fairness assumption $\mathcal{F}$ for TS and a *safety* property P, if $TS \models_{\mathcal{F}} P$ then $TS \models P$ (the other direction being true for any P).